

Two-nucleon scattering in a modified Weinberg approach with a symmetry-preserving regularization

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Abstract

We consider the nucleon-nucleon scattering problem by applying time-ordered perturbation theory to the Lorentz invariant formulation of baryon chiral perturbation theory. Using a symmetry preserving higher derivative form of the effective Lagrangian, we exploit the freedom of the choice of the renormalization condition and obtain an integral equation for the scattering amplitude with an improved ultraviolet behavior. The resulting formulation is used to quantify finite regulator artifacts in two-nucleon phase shifts as well as in the chiral extrapolations of the S-wave scattering lengths and the deuteron binding energy. This approach can be straightforwardly extended to analyze few-nucleon systems and processes involving external electroweak sources.

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I. INTRODUCTION

The effective field theory (EFT) approach to the strong interaction at low energies is a perturbative framework based on an expansion in terms of small energy and/or masses divided by some large scale. Higher orders of these small parameters are suppressed at low energies thus leading to a reliable order-by-order calculation of physical quantities (see e.g. Refs. [1–4] for reviews). An underlying assumption of this approach is that quantum chromodynamics (QCD) is the correct theory of the strong interaction. That is, the S -matrix of QCD correctly reproduces the properties of the strong interaction. Based on the symmetries of QCD, EFT aims at reproducing the S -matrix of QCD in the low-energy region. Poles corresponding to one-particle hadronic states are represented in an EFT by dynamical fields, branch points are generated by quantum corrections (i.e. by loop diagrams). Interaction terms of the effective Lagrangian generate tree-level diagrams and also contribute in loop graphs. Some of the interactions leading to contributions suppressed due to their higher order in the chiral expansion at tree level can lead to severely divergent contributions in loop diagrams. However, these large contributions can be removed by appropriate renormalization. This program works without complication in the mesonic sector. Care has to be taken in the single-nucleon sector due to the non-vanishing chiral limit of the nucleon mass. The issue turns out to be highly non-trivial for few-nucleon systems, first considered in the framework of chiral EFT by Weinberg [5]. A complication emerges from the nonperturbative nature of the problem at hand which requires infinitely many loop diagrams to be summed up already at leading order (LO). This can be achieved by defining an effective potential and solving the Lippmann-Schwinger (LS) equation for the amplitude. Renormalization of the solutions of such integral equations is a non-trivial task. It led to much controversy and different formulations of the chiral expansion in the few-nucleon sector [6–33].

In the standard non-relativistic formulation of chiral EFT, the LO LS equation is well known to be linearly divergent due to the singular nature of the tensor part of the static one-pion exchange potential (OPEP). Accordingly, an iterative solution of the LS equation in each spin-triplet nucleon-nucleon (NN) partial wave requires the inclusion of an infinite number of counter terms to absorb all emerging ultraviolet (UV) divergences. Clearly, this is not feasible in practice. The commonly used approach for dealing with this issue is to introduce a finite UV cutoff, whose value has to be taken of the order of the hard scale of the problem [12, 16, 34, 35]. In practice, the range of such momentum cutoffs turns out to be rather limited from above due to the appearance of spurious deeply bound states, which provide a severe (technical) complication for applications beyond the NN system.

Recently, an alternative approach to chiral EFT for NN scattering has been formulated [27], which employs the Lorentz invariant Lagrangian and makes use of time-ordered perturbation theory (TOPT). Contrary to the nonrelativistic formulation, it leads to a renormalizable LO integral equation. Accordingly, the cutoff parameter can be varied from the hard scale of the problem to infinity. To achieve renormalizability beyond LO, higher-order corrections to the amplitude have to be included perturbatively.¹ This novel cutoff-independent approach has already been applied to calculate neutron-proton phase shifts [27], perform chiral extrapolations of the S-wave scattering length and the deuteron binding energy [37] and to analyze the electromagnetic form factors of the deuteron [32] at LO in the chiral expan-

¹ Higher-order short-range terms can also be included non-perturbatively, see Ref. [36] for a calculation of the neutron-proton 1S_0 phase shift with the subleading contact term being treated non-perturbatively.

sion. Derivation of the subleading corrections to the NN potential is in progress. For recent extensions to $D\bar{D}^*$ meson scattering and strangeness $S = -1$ hyperon-nucleon scattering see Refs. [38] and [39], respectively.

As already pointed out, when extending these calculations to higher orders in the chiral expansion, the corrections to the potential need to be treated perturbatively if explicit renormalizability of the scattering amplitude is to be maintained. From the practical point of view, it is, however, advantageous to treat the whole potential including higher-order contributions non-perturbatively when solving the corresponding integral equations. In this case, the general arguments of Ref. [12, 16, 34, 35] apply and the cutoff should not be taken beyond the hard scale of the problem. While conceptually equivalent to the standard non-relativistic chiral EFT with finite cutoff, the Lorentz invariant approach with non-perturbative treatment of the potential beyond the LO is expected to offer more flexibility when choosing the cutoff range.

In this paper we describe in detail the introduction of a symmetry-preserving higher-derivative regularization using the chiral EFT formulation for NN scattering of Ref. [27], see also Refs. [40–42] for a related earlier work along similar lines.² The main idea of the approach can be explained as follows. According to the general formalism of chiral EFT, one has to include *all* terms in the effective Lagrangian which are consistent with the underlying symmetries of QCD. Some of these terms are redundant in the sense that one can get rid off them by using field redefinitions. Doing so one usually reduces the effective Lagrangian to its minimal form. While convenient, this step is, however, by no means necessary. One can perform calculations using the original truly most general effective Lagrangian, or any other “non-minimal” form obtained by applying specifically chosen field redefinitions. Moreover, in certain cases, it turns out to be more convenient to exploit this freedom of choosing different forms of the effective Lagrangian. While field transformations do not change physical observables when treated exactly, they correspond to re-summation of certain higher order contributions in perturbative calculations. In the context of low-energy EFT, this freedom can be exploited to introduce a symmetry-preserving regularization [40, 41], which, unlike dimensional regularization, is also applicable beyond standard perturbation theory. We use TOPT and define the effective potential as a sum of all two-particle irreducible diagrams contributing in the scattering amplitude. The NN scattering amplitude is obtained by solving the corresponding integral equation. Compared to the work of Ref. [27], the current approach allows one to treat higher-order corrections to the potential non-perturbatively which has practical advantages when applying this scheme to three- and more-nucleon systems and reactions involving external electroweak probes.

As an application, we use the resulting formulation to calculate neutron-proton phase shifts and perform chiral extrapolations of NN S-wave scattering lengths and the deuteron binding energy at LO. The employed higher-derivative regularization scheme allows us to quantify finite-regulator artifacts in the calculated observables, see Refs. [45, 46] for a related discussion. Our results provide useful information on the size of the neglected higher-order terms and are confronted with the findings of Ref. [47], where a different strategy was used to estimate the theoretical uncertainty.

Our work is organized as follows: in section II we consider the effective Lagrangian and TOPT while the integral equation for the NN scattering amplitude is derived in section III. Next, in section IV, we calculate the NN phase shifts at LO and study the dependence of

² Our framework differs from the “semi-relativistic” scheme of Ref. [42] by the usage of TOPT [43, 44].

the chiral extrapolations for S-wave scattering lengths and the deuteron binding energy on the regulator. Finally, the results of our work are summarized in section V.

II. FORMALISM

As already pointed out in the introduction, we employ the manifestly Lorentz invariant effective Lagrangian for pions and nucleons. It includes the purely mesonic, the πN , NN , ... parts,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\pi + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \dots \quad (1)$$

The effective Lagrangian is organised as an expansion in powers of the quark-masses and derivatives. The lowest-order mesonic Lagrangian reads [48]³

$$\mathcal{L}_2 = \frac{F^2}{4} \text{Tr} \left[\partial_\mu U (\partial^\mu U)^\dagger \right] + \frac{F^2 M^2}{4} \text{Tr} (U^\dagger + U), \quad (2)$$

where U is a unimodular unitary (2×2) matrix containing the Goldstone boson fields, F denotes the pion-decay constant in the chiral limit: $F_\pi = F[1 + \mathcal{O}(m_q)] = 92.2$ MeV. We consider the isospin-symmetric limit $m_u = m_d = m_q$. The lowest-order expression for the squared pion mass is $M^2 = 2Bm_q$, where B is related to the quark condensate $\langle \bar{q}q \rangle_0$ in the chiral limit [48].

The lowest-order Lagrangian of the one-nucleon sector is given as [49]

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left(i\gamma_\mu D^\mu - m + \frac{1}{2} \overset{\circ}{g}_A \gamma_\mu \gamma_5 u^\mu \right) \Psi, \quad (3)$$

where the nucleons are represented by two four-component Dirac fields $\Psi = (p, n)^T$, m and $\overset{\circ}{g}_A$ stand for the nucleon mass and the axial-vector coupling constant in the chiral limit, respectively, while $D_\mu \Psi = (\partial_\mu + \Gamma_\mu) \Psi$ denotes the covariant derivative. The quantities u , u_μ and Γ_μ are given by

$$u^2 = U, \quad u_\mu = iu^\dagger \partial_\mu U u^\dagger, \quad \Gamma_\mu = \frac{1}{2} [u^\dagger \partial_\mu u + u \partial_\mu u^\dagger]. \quad (4)$$

To generate an undressed propagator with an improved ultraviolet behavior as compared to that of the Lagrangian of Eq. (1), we consider an effective Lagrangian with additional terms [41]. In particular, by adding *symmetry-preserving* terms to the standard BChPT Lagrangian of Eq. (1), we can obtain the following modified nucleon propagator

$$S_N^\Lambda(p) = \frac{1}{(\not{p} - m + i0^+)} \prod_{j=1}^{N_\Psi} \frac{\Lambda_{\Psi j}^2}{\Lambda_{\Psi j}^2 + m^2 - p^2 - i0^+}. \quad (5)$$

Here, the $\Lambda_{\Psi j}$ are, in general, independent parameters.

³ For the purposes of the current work we switch off the external sources except of course the scalar one that accounts for the explicit symmetry breaking.

The choice of the additional terms of the Lagrangian leading to Eq. (5) is not unique. Following Ref. [41] for the additional terms of the Lagrangian we choose

$$\mathcal{L}_{\pi N}^{\text{reg}} = \sum_{n=1}^{N_\Psi} \frac{Y_n}{2} [\bar{\Psi} (i\gamma_\mu D^\mu - m) (D^2 + m^2)^n \Psi + h.c.], \quad (6)$$

where the Y_n are functions of $\Lambda_{\Psi j}$. For example, for the modified nucleon propagator

$$S_N^\Lambda(p) = \frac{\Lambda^2}{(\not{p} - m + i0^+) (\Lambda^2 + m^2 - p^2 - i0^+)} \quad (7)$$

we have $N_\Psi = 1$ and $Y_1 = 1/\Lambda^2$.

Depending on the order of the performed calculations we can choose the modified propagators such that all loop Feynman diagrams contributing to the given order converge. The additional higher-derivative terms introduced in the effective Lagrangian can be re-expressed in a canonical form by using field transformation. This makes clear that any Λ dependence of physical quantities can be absorbed in the redefinition of the parameters of the standard effective Lagrangian.

The effective NN Lagrangian contains terms with different numbers of derivatives acting on the nucleon field Ψ . Using field redefinitions and re-organizing the terms one can achieve that the effective Lagrangian contributing to the LO NN potential contains only terms without derivatives:

$$\begin{aligned} \mathcal{L}_{\text{NN}} = & C_S^a \bar{\Psi} \tau^a \Psi \bar{\Psi} \tau^a \Psi + C_T^a \bar{\Psi} \tau^a \sigma_{\mu\nu} \Psi \bar{\Psi} \tau^a \sigma^{\mu\nu} \Psi + C_{AV}^a \bar{\Psi} \tau^a \gamma_5 \gamma_\mu \Psi \bar{\Psi} \tau^a \gamma_5 \gamma^\mu \Psi \\ & + C_V^a \bar{\Psi} \tau^a \gamma_\mu \Psi \bar{\Psi} \tau^a \gamma^\mu \Psi, \end{aligned} \quad (8)$$

where summation over $a = 0, 1, 2, 3$ is implied, with τ^0 denoting the unit matrix and τ^i ($i = 1, 2, 3$) referring to the Pauli (isospin) matrices. Further, $C_I^1 = C_I^2 = C_I^3$ for all I . Note that the LO effective Lagrangian of Eq. (8) also contributes to the potential at higher orders. For the most general Lorentz invariant NN Lagrangian of the second order (in small momentum and quark masses) see Ref. [50].

For the purposes of the current work it is convenient to take the additional terms of the following form

$$\begin{aligned} \mathcal{L}_{\pi N}^{\text{reg}} = & \frac{1}{2\Lambda^2} [\bar{\Psi} (i\gamma_\mu D^\mu - m) (D^2 + m^2) \Psi + h.c.] \\ \equiv & -\frac{1}{2\Lambda^2} [\bar{\Psi} (i\gamma_\mu D^\mu - m) \vec{D}^2 \Psi + h.c.] + \frac{1}{2\Lambda^2} [\bar{\Psi} (i\gamma_\mu D^\mu - m) (D_0^2 + m^2) \Psi + h.c.]. \end{aligned} \quad (9)$$

Here and in the following, we include the contribution of the first term of Eq. (9) to the nucleon two-point function non-perturbatively, which affects the form of the propagator, while the second term is treated in perturbation theory. The corresponding propagator has the form

$$S_N^\Lambda(p) = \frac{\Lambda^2}{(\not{p} - m + i\epsilon) (\Lambda^2 + \vec{p}^2)}. \quad (10)$$

To obtain the rules of the TOPT we first draw all Feynmann diagrams, relevant for the given process (in principle an infinite number of them), assign the momenta to propagator lines and perform the trivial integrations using the delta functions appearing at the vertices.

The remaining overall delta function ensures the momentum conservation for the external legs of diagrams. Next we perform integrations over the zeroth components of the loop integration variables. As a result we are led to the diagrams of the TOPT [43, 44]. The same time-ordered diagrams can be obtained directly using the following rules of the TOPT:⁴

The S matrix for a transition $\alpha \rightarrow \beta$ may be written as

$$S_{\beta\alpha} = \delta_{\beta\alpha} - (2\pi)^4 i M_{\beta\alpha} \delta^4(P_\beta - P_\alpha) \Pi_n^{\alpha,\beta} (2\pi)^{-3/2} (2\omega_n)^{-1/2}, \quad (11)$$

where P^μ is the total four-momentum, $\omega_n = (\vec{p}_n^2 + m_n^2)^{1/2}$, where \vec{p}_n is the three-momentum of a particle with mass m_n and $\Pi_n^{\alpha,\beta}$ stands for the product over all particles in the initial and final states. The invariant amplitude M is obtained by using the following diagrammatic rules:

- Draw all possible time-ordered diagrams for the transition $\alpha \rightarrow \beta$. That is, draw each Feynmann diagram with N vertices $N!$ times while ordering the vertices in every possible way in a sequence running from right to left. Label each line with a four dimensional momentum $p = (p_0, \vec{p})$ as prescribed by the corresponding Feynmann diagram.

- Include a factor

$$(2\pi)^{-3} (2\omega)^{-1} \quad (12)$$

for every internal line. For fermion lines, include also factors $(\not{p} + m)/(\vec{p}^2 + \Lambda^2)$.

- Multiply the expressions with the coupling constants and include the relevant factors such as momenta, γ -matrices etc. for every vertex. For each time-ordered diagram, the zeroth component of the integration variable in the numerators of fermion propagators is to be replaced by the energy of the corresponding fermion line. The sign of this energy is positive for a particle line and negative for an antiparticle line. Care has to be taken also of zeroth components present in interaction vertices, that is, one needs to identify the poles in the complex plane of the zeroth components of the integration momenta, which have been picked up to obtain the given TOPT diagram, and substitute the corresponding expressions or the zeroth components of the momenta in the vertices.
- For every intermediate state γ , i.e. a set of lines between any of two vertices, include an energy denominator

$$[E_\alpha - E_\gamma + i\epsilon]^{-1}, \quad (13)$$

where $E = \sum \omega$ is the total energy of the state.

- Integrate the product of these factors over all internal (three) momenta, and sum the result over all diagrams.

⁴ As our EFT Lagrangian contains fermion fields and the interaction terms with time derivatives, we need to keep track of the poles for which the residues have been picked up.

III. INTEGRAL EQUATION FOR THE OFF-SHELL SCATTERING AMPLITUDE

The N-nucleon scattering amplitude M_{2N} is obtained from 2N-point vertex function $\tilde{\Gamma}_{2N}$ by applying the standard LSZ formula

$$M_{2N} = Z_\psi^N \bar{u}_{out1} \dots \bar{u}_{outN} \tilde{\Gamma}_{2N} u_{in1} \dots u_{inN}, \quad (14)$$

where Z_ψ is the residue of the nucleon propagator and u_{inI} , \bar{u}_{outJ} are Dirac spinors of the incoming and outgoing nucleons. The vertex function with on-shell momenta is given as a sum of an infinite number of TOPT diagrams.

For nucleon-nucleon scattering, the purely two-nucleon intermediate states are enhanced [5]. Therefore it is convenient to define effective potentials as the sums of all two-particle irreducible TOPT diagrams. The on-shell four-point vertex function $\tilde{\Gamma}$ is then given by an infinite series

$$\begin{aligned} \tilde{\Gamma} &= \tilde{V} + \bar{V}G\bar{V} + \bar{V}GVG\bar{V} + \bar{V}GVGVG\bar{V} + \dots \\ &= \tilde{V} + \bar{V}G\bar{V} + \bar{V}G[V + VGV + \dots]G\bar{V} = \tilde{V} + \bar{V}G\bar{V} + \bar{V}G\Gamma G\bar{V}, \end{aligned} \quad (15)$$

where G is the two-nucleon Greens function and $\tilde{\Gamma}$, Γ , \tilde{V} , \bar{V} and V are the on-shell vertex function, the off-shell vertex function, the on-shell potential, the half-off shell potential and the off-shell potential, respectively. Note here that we include the numerators of the fermion propagators in the definition of the potential so that G contains a factor of $2m$ for each nucleon. The off-shell vertex function Γ can be obtained by solving the following equation:

$$\Gamma = V + V\Gamma. \quad (16)$$

We define a projected potential

$$V_P = P_+ P_+ V P_+ P_+, \quad (17)$$

where

$$P_+ = \frac{1 + \not{v}}{2}, \quad v = (1, 0, 0, 0), \quad (18)$$

and assume that it can be expanded in some generic parameter according to

$$V_P = V_{P0} + V_{P1} + \dots. \quad (19)$$

Next, we introduce the corresponding vertex function Γ_0 as the solution to the following equation

$$\Gamma_0 = V_{P0} + V_{P0} G \Gamma_0. \quad (20)$$

It is easily seen from Eq. (20) that

$$\Gamma_0 = \Gamma_{P0} \equiv P_+ \dots P_+ \Gamma_0 P_+ \dots P_+. \quad (21)$$

Within our approach we identify the reduced LO potential V_{P0} as the LO effective potential V_0 and expand the effective potential as

$$V \equiv V_0 + (V - V_0) = V_0 + V_1 + V_2 + \dots. \quad (22)$$

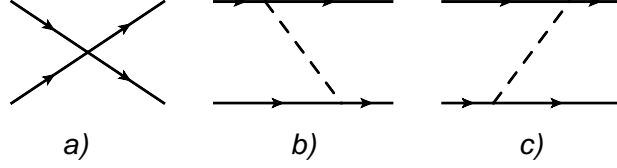


FIG. 1: Time-ordered diagrams contributing to the LO NN potential.

We treat V_0 non-perturbatively, i.e. we calculate the LO vertex function Γ_0 from Eq. (20) and include the corrections perturbatively. The NLO vertex function Γ_1 is given as

$$\Gamma_1 = V_1 + V_1 G \Gamma_0 + \Gamma_0 G V_1 + \Gamma_0 G V_1 G \Gamma_0. \quad (23)$$

It is straightforward to express the further higher-order corrections to the vertex function in terms of the potentials of the corresponding orders and vertex functions of lower orders. Further, when calculating the effective potential, we write the numerator of the standard fermion propagator as

$$\not{p} + m = 2m P_+ + (\not{p} - m \not{v}), \quad (24)$$

and identify the second term as a higher-order correction.

To determine the physical amplitude order by order in an expansion in terms of small parameters we rewrite the Dirac spinors as

$$\begin{aligned} u &= \left(1 + \frac{\not{p} - p \cdot v}{m + p \cdot v}\right) P_+ u(p) = u_0 + u_1. \\ \bar{u} &= \bar{u}(p) P_+ \left(1 + \frac{\not{p} - p \cdot v}{m + p \cdot v}\right) = \bar{u}_0 + \bar{u}_1. \end{aligned} \quad (25)$$

For small three-momenta of the nucleons, we identify u_1 and \bar{u}_1 as corrections suppressed compared to the dominant u_0 and \bar{u}_0 terms.

For two-nucleon scattering, we calculate the scattering amplitude in the center-of-mass frame. The vertex function $\Gamma(E, \vec{p}', \vec{p})$ satisfies the equation (the spin and isospin indices are suppressed)

$$\begin{aligned} \Gamma(E, \vec{p}', \vec{p}) &= V(E, \vec{p}', \vec{p}) - \frac{m^2}{2} \int \frac{d^3 \vec{k}}{(2\pi)^3} V(E, \vec{p}', \vec{k}) G(E, k, \Lambda) \Gamma(E, \vec{k}, \vec{p}), \\ G(E, k, \Lambda) &= \frac{\Lambda^4}{(\vec{k}^2 + \Lambda^2)^2 (\vec{k}^2 + m^2) (E/2 - \sqrt{\vec{k}^2 + m^2} + i\epsilon)}. \end{aligned} \quad (26)$$

Here, \vec{p} and \vec{p}' are the three-momenta of the incoming and outgoing nucleons, respectively, and $E = 2\sqrt{\vec{q}^2 + m^2}$ is the energy of the two nucleons. In the partial wave basis, Eq. (26) leads to the following equations with the partial wave projected potential $V_{\nu l}^{sj}(p', p)$,

$$T_{\nu l}^{sj}(E, p', p) = V_{\nu l}^{sj}(E, p', p) + \frac{m^2}{2} \sum_{\nu'} \int_0^\infty \frac{dk k^2}{(2\pi)^3} V_{\nu \nu'}^{sj}(E, p', k) G(E, k, \Lambda) T_{\nu' l}^{sj}(E, k, p). \quad (27)$$

A standard UV counting shows that in the limit of large integration momenta, Eq. (26) has a milder UV behavior than the corresponding LS equation. That is, its solutions are expected to show less sensitivity to the variation of the finite values of the parameter Λ .

IV. APPLICATIONS TO NUCLEON-NUCLEON SCATTERING AT LEADING ORDER

The LO effective potential for NN scattering, $V_0 = V_{0,C} + V_{0,\pi}$, consists of the contact interaction part and two one-pion exchange time-ordered diagrams shown in Fig. 1. The projected OPEP has the form

$$V_\pi(\vec{p}', \vec{p}) = -\frac{g_A^2}{4F^2} \frac{\vec{\tau}_1 \cdot \vec{\tau}_2}{\sqrt{(\vec{p} - \vec{p}')^2 + M^2}} \times \frac{[\vec{\sigma}_1 \cdot (\vec{p} - \vec{p}')] [\vec{\sigma}_2 \cdot (\vec{p} - \vec{p}')] }{\sqrt{(\vec{p} - \vec{p}')^2 + M^2} + \sqrt{\vec{p}^2 + m^2} + \sqrt{\vec{p}'^2 + m^2} - E - i\epsilon}. \quad (28)$$

In the calculations of the phase shifts described below, we approximate the two-nucleon energy E in Eq. (28) by $2m$.

We solve the integral equation (27) using standard numerical methods. For the various parameters, we employ the values of $M_\pi = 138.0$ MeV, $F_\pi = 92.2$ MeV and $m = 938.9$ MeV. Further, for the nucleon axial-vector coupling, we use the effective value $g_A = 1.285$ which takes into account the Goldberger-Treiman discrepancy. Two linear combinations of low-energy constants $C_S + C_V$ and $C_{AV} + 2C_T$ contributing at LO [42] are adjusted to reproduce the 1S_0 and 3S_1 phase shifts from the Nijmegen partial wave analysis [51] below $E_{\text{lab}} = 50$ MeV.

In Fig. 2, we show our results for the neutron-proton S-, P- and D-waves and the mixing angles ϵ_1 and ϵ_2 for the regulator values of the order of the expected breakdown scale in the problem, namely $\Lambda = 600, 800, 1000$ and 1200 MeV, in comparison with the Nijmegen [51] and the GWU single-energy partial wave analysis (PWA) [52]. We observe a strong Λ -dependence in the 3P_0 partial wave caused by the presence of a pole in the complex plane on the unphysical sheet. To get the phase shifts closer to the PWA we have included the NLO contact interaction.⁵ As the OPEP is strongly attractive in this channel, the introduced counter term has to be repulsive. Notice that for the employed form of the integral equation and higher-derivative regularization, we would have to take an infinitely large repulsive value for the counter term already for $\Lambda \sim 566$ MeV to reproduce the low-energy behavior of the phase shift. For larger values of Λ , the low energy phase shifts can be reproduced by adjusting an *attractive* counter term, which, however, leads to the appearance of a deeply bound state in this channel. We also show in Fig. 2 the 3P_0 phase shifts which result from taking an infinitely large repulsive value for the counter term instead of tuning it to the Nijmegen PWA, which prevents the appearance of deeply-bound states in the considered range of Λ .

For the sake of comparison, we also show in Fig. 2 the results from Ref. [27] which correspond to the limit of $\Lambda \rightarrow \infty$. As already pointed out in the introduction, such a limit can be taken in the LO equation, but does not represent a legitimate procedure for a

⁵ We remind the reader that the contributions of a given term in the effective Lagrangian to observables depend on the employed scheme and the choice of renormalization conditions. The results of Refs. [33, 47] based on the nonrelativistic framework and utilizing a coordinate-space regulator chosen in the range of $R = 0.8 \dots 1.2$ fm do not support the need to depart from Weinberg's original power counting for contact interactions, see also [53] for a related discussion.

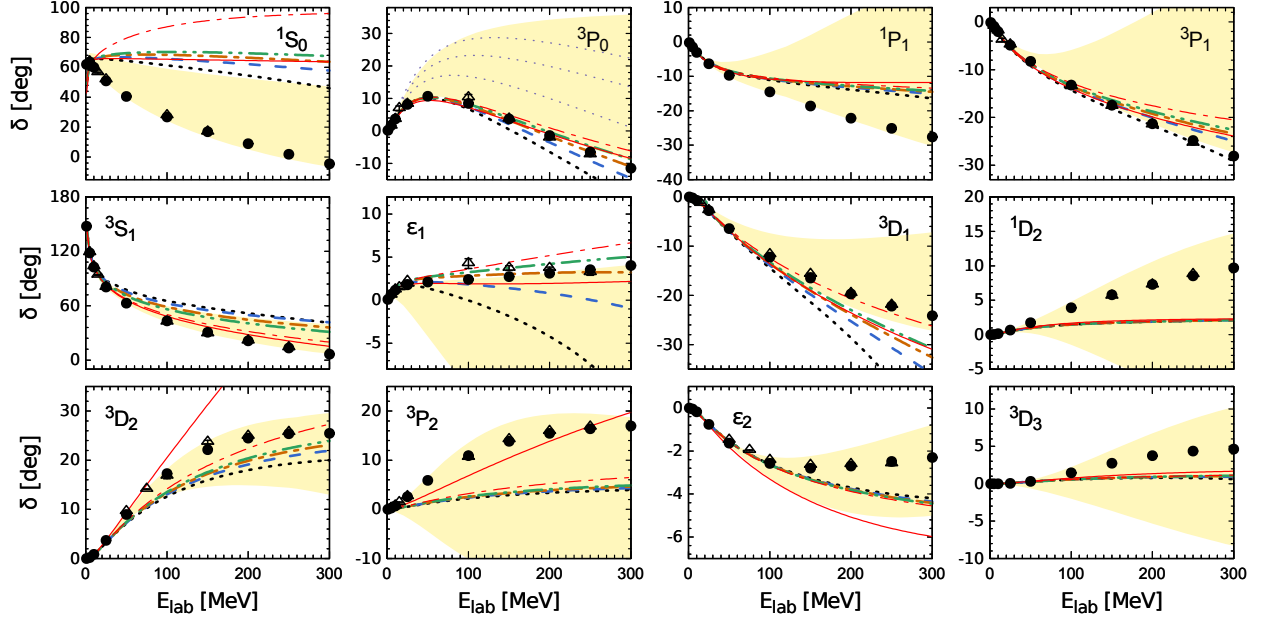


FIG. 2: (Color online) Phase shifts and mixing angles calculated at LO as functions of laboratory energy in comparison with the Nijmegen [51] (filled circles) and GWU single-energy partial wave analyses [52] (open triangles). Thick black dotted, blue dashed, brown dashed-dotted and green dashed-double-dotted lines correspond to $\Lambda = 600, 800, 1000$ and 1200 MeV, respectively. In the 3P_0 channel, the three upper curves shown by thin violet dotted lines show the predictions including the infinitely strong repulsive contact interaction for $\Lambda = 800, 1000$ and 1200 MeV (from bottom to top). Thin red dashed-dotted lines give the result for $\Lambda \rightarrow \infty$, while thin solid lines show the LO results from Ref. [27] obtained in the limit of $\Lambda \rightarrow \infty$ by employing the *static* form of the one-pion exchange potential. Light shaded areas depict the estimated theoretical uncertainty from the truncation of the chiral expansion at LO from Ref. [47] for the regulator choice of $R = 0.9$ fm.

non-perturbative inclusion of higher-order corrections. Notice that our finite- Λ predictions are, strictly speaking, not expected to converge against the infinite- Λ results of Ref. [27] shown by the thin solid lines due to the different form of the employed one-pion exchange potential. In particular, a static approximation of the nonstatic expression for the OPEP in Eq. (28) was employed in that work. Clearly, both choices for the OPEP are valid as the difference between the two forms is of a higher order. Our infinite- Λ predictions based on the non-static form of the OPEP given in Eq. (28) are shown by the thin dashed-dotted lines in Fig. 2.

We are now in the position to discuss the implications of our findings in connection with the estimation of the theoretical uncertainty at LO. To this aim, we show by the light-shaded areas the LO predictions of the new generation of chiral NN potentials of Ref. [47] obtained using a semi-local regularization within the nonrelativistic framework, along with the estimated uncertainty from the truncation of the chiral expansion, see that reference for more details. We restrict ourselves to showing the results corresponding to the regulator choice of $R = 0.9$ fm found to lead to the most accurate predictions. Notice that the theoretical uncertainty was quantified in that work by estimating the size of higher-order contributions to the potential *without* relying on the variation of the regulator R . Thus, the results of our study utilizing a large variation of the cutoff Λ in the range from 600 MeV to

∞ provide an excellent opportunity to test the reliability of the uncertainty quantification approach formulated in Ref. [47].

It is reassuring to see that our predictions for different values of Λ shown by various lines in Fig. 2 lie within the estimated theoretical uncertainty in almost all considered cases. The most notable exception is the 1S_0 phase shift, for which we do observe a somewhat larger deviation from the Nijmegen PWA for $\Lambda = \infty$ as compared with the results of Ref. [47]. Notice, however, that the deviations between our results for $\Lambda = 600 \dots 1200$ MeV from the Nijmegen phase shifts are comparable to the width of the uncertainty band. It is well known that the one-pion exchange potential only generates about a half of the effective range in this channel, see e.g. Ref. [54], and that the contribution of the subleading contact interaction is large [36]. These features are, in fact, reflected in the rather large estimated theoretical uncertainty in this channel. The dominant role of short-range interactions in this partial wave is also consistent with by far the slowest observed convergence of the calculated phase shifts with respect to Λ . Another quantity which appears to be rather sensitive to short-range physics is the mixing angle ϵ_1 . This observation is consistent with results obtained within different approaches, see e.g. Refs. [36, 55, 56].

Our results also illustrate, that the assessment of the theoretical uncertainty based on a cutoff variation alone should be taken with care as it is insensitive to neglected long-range interactions. In particular, for the nonstatic OPEP in Eq. (28), we obtain nearly cutoff-independent results for the 1P_1 , 1D_2 , 3P_2 and 3D_3 phase shifts and the mixing angle ϵ_2 over the range of $600 \text{ MeV} < \Lambda < \infty$, which, however, deviate from the empirical phase shifts. This lets one conclude that the discrepancies with the Nijmegen PWA in these channels are largely driven by higher-order long-range interactions. Employing the two different forms of the OPEP which are equivalent at LO allows us to probe sensitivity to certain kinds of higher-order long-range contributions and leads to a more realistic uncertainty estimation in the 3P_2 partial wave and the mixing angle ϵ_2 . Generally, the results of the present work fit well with those of the nonrelativistic calculation in Ref. [36] and show no contradictions with the theoretical uncertainty at LO estimated in that paper.

As a second application, we consider the quark- or, equivalently, pion-mass dependence of the neutron-proton S-wave scattering lengths a_{1S_0} , a_{3S_1} and the deuteron binding energy. At LO in the modified Weinberg approach, the M_π -dependence of the scattering amplitude emerges entirely from the explicit M_π -dependence of the pion propagator in the OPEP and, therefore, can be predicted in a parameter-free way. In Ref. [37], we have shown the predicted M_π -dependence of $a_{1S_0}^{-1}$, $a_{3S_1}^{-1}$ and the deuteron binding energy E for $\Lambda \rightarrow \infty$ using the static form of the OPEP. For the spin-triplet channel, the predicted M_π -dependence suggests a weaker attraction for larger-than-physical values of the quark masses in agreement with the results of Refs. [57, 58]. On the other hand, lattice QCD calculations for pion masses $M_\pi > 300$ MeV indicate an opposite trend with the deuteron being stronger bound [59–63], except for the results by HAL QCD collaboration which finds no bound deuteron for $M_\pi = 469 \dots 1171$ MeV [64]. For a summary of the current status of lattice QCD results and a related discussion based on low-energy theorems for NN scattering see Refs. [54, 65]. Thus, it is important to quantify the theoretical uncertainty of the LO prediction of Ref. [37]. To this aim, we calculate the quark mass dependence of $a_{1S_0}^{-1}$, $a_{3S_1}^{-1}$ and E at LO using the higher-derivative regularization framework for different values of Λ as visualized in Fig. 3. Here and in what follows, we work in the isospin limit and employ the LO approximation to relate the pion and the average light-quark masses, M_π and m_q , namely, $M_\pi^2 = 2Bm_q$. Furthermore, in all cases discussed below, we tune the contact interaction to

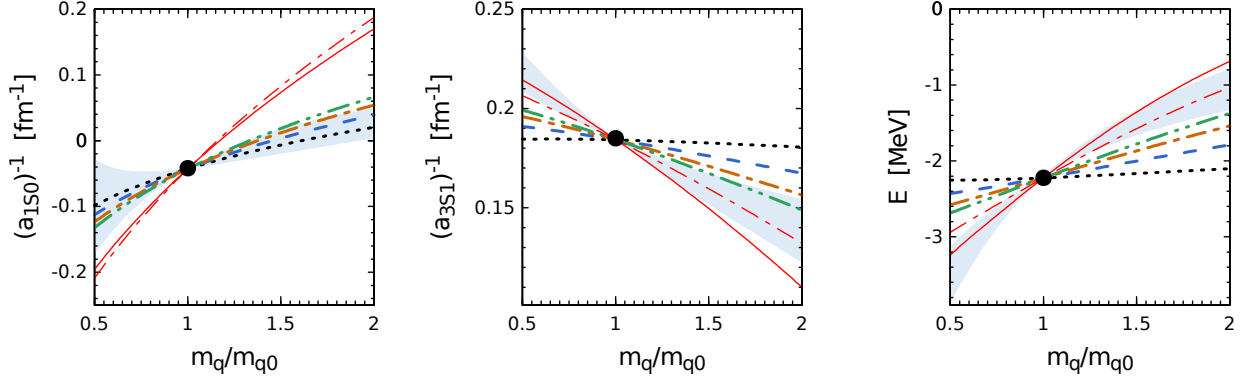


FIG. 3: (Color online) Quark mass dependence of the inverse scattering lengths $a_{1S_0}^{-1}$, $a_{3S_1}^{-1}$ and the deuteron binding energy E at LO. Solid dots show the empirical values for the considered observables at the physical value m_{q0} of the quark mass while light-shaded bands give the N²LO results from Ref. [58] based on the resonance saturation hypothesis for short-range operators. For remaining notation see Fig. 2.

exactly reproduce the corresponding observable at the physical point.

For all considered quantities, the difference in the predictions based on the static and nonstatic forms of the OPEP in the infinite- Λ limit turns out to be small showing that the corresponding higher-order long-range terms in the potential play a minor role. On the other hand, we do observe a significant Λ -dependence indicating that the contributions of the neglected quark-mass dependent contact interactions are substantial, see also [58, 66–69] for similar conclusions achieved using different approaches. For the spin-singlet channel, the resulting Λ -dependence of the chiral extrapolation of the scattering length for unphysical quark masses shows a similar pattern to the scattering phase shift at the physical value of the quark mass. In particular, we observe a very slow convergence with respect to the cutoff Λ . Interestingly, for a_{1S_0} , our results for Λ in the range of $\Lambda = 600 \dots 1200$ MeV appear to be in a very good agreement with the predictions of Ref. [58] obtained within the standard nonrelativistic formulation at N²LO in the chiral expansion. In that work, the M_π -dependence of the short-range operators was estimated via the resonance saturation hypothesis by invoking unitarized chiral perturbation theory in combination with lattice QCD results to describe the dependence of the resonance positions on the quark mass. The light-shaded bands shown in Fig. 3 correspond to cutoff variation over a certain range, see [58] for more details, but do not take into account the uncertainty associated with the usage of the resonance saturation hypothesis. Interpreting the spread between the various lines in Fig. 3 as an estimation of the contributions from higher order M_π -dependent contact interactions, we conclude that the LO predictions cannot exclude the possibility of an increasing attraction in the 3S_1 channel for unphysically large values of M_π reported in the recent lattice QCD calculations. Notice, however, that the preferred scenario based on our LO calculations still corresponds to decreasing of the deuteron binding energy with increasing values of the pion mass near the physical point.

V. SUMMARY

We considered the nucleon-nucleon scattering problem in the framework of higher derivative formulation of BChPT of Ref. [41] by applying the TOPT to the Lorentz invariant effective Lagrangian. The formulation preserves all underlying symmetries and is therefore well suited for studying processes involving external electroweak probes and chiral extrapolations. At the same time, it leads to non-singular integral equations for the NN scattering amplitude at any given order in the chiral expansion and, differently to the framework of Ref. [27], allows for a non-perturbative inclusion of higher-order contributions to the potential. This feature may be particularly useful for facilitating the generalization of this approach to three- and more-nucleon systems and is achieved by making use of the freedom in the choice of the form of the effective Lagrangian or, equivalently, renormalization conditions parametrized in terms of the scale Λ . Given that the LO equation for the NN scattering amplitude is renormalizable in our scheme, the parameter Λ can be varied over a large range or even completely eliminated yielding the approach of Ref. [27]. On the other hand, if higher-order contributions to the potential are to be taken into account non-perturbatively, the parameter Λ needs to be chosen of the order of the expected breakdown scale of the theory such as e.g. the mass of the ρ meson.

We have applied this framework to test the estimation of the theoretical uncertainty for NN phase shifts and mixing angles of Ref. [47] at LO and to quantify the accuracy of the chiral extrapolations of the S-wave scattering lengths a_{1S_0} , a_{3S_1} and the deuteron binding energy E predicted in Ref. [37]. The resulting sensitivity of the NN scattering observables to the considered variation of the parameter Λ over a large range is found to be consistent with the LO uncertainty bands of Ref. [47] generated without relying on cutoff variation. For the chiral extrapolations of a_{1S_0} , a_{3S_1} and E , we find a rather sizable sensitivity to neglected higher-order M_π -dependent contact interactions which, however, should not come as a surprise given the strongly fine-tuned nature of these quantities, also noted in earlier work [58, 66–69].

Acknowledgments

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